REVIEWS

Les Principes de l'Analyse Dimensionelle. By René Saint-Guilhem. Gauthier-Villars, 1962. 77 pp.

Similarity and Dimensional Methods in Mechanics. By L. I. Sedov. (Translated from 4th Russian edition by M. Friedman, and edited by M. Holt.) Academic Press, 1959. 363 pp. $14.00.

The subject of similitude in fluid mechanics is like an onion: peel off one layer and you may find another. At least four layers appear, for example, in the familiar problem of incompressible viscous flow over a flat plate. First, dimensional analysis reduces the physical parameters to a single dimensionless combination: the Reynolds number based on length. Secondly, Prandtl's boundary-layer approximation removes the dependence upon Reynolds number from the problem. Thirdly, the absence of a characteristic geometric length for the semi-infinite plate, or for the finite plate ahead of its trailing edge (in view of the parabolic nature of the boundary-layer equations) permits reduction of the partial differential equations to an ordinary one. Finally, the awkward two-point boundary-value problem can be replaced by an initial-value problem using a group property of the differential equation due to Töpfer.

These are typical applications of a more-or-less related group of concepts that permeate the theoretical literature of fluid mechanics: dimensional analysis, 'inspectional analysis', homogeneity, modelling, analogues, similarity rules, group invariance, self-similar solutions. The boundaries between these ideas are not clear, and no unified treatment of them is available. The need is most nearly met by Birkhoff's provocative (and, to some readers, infuriating) book on *Hydrodynamics—a Study in Logic, Fact, and Similitude*. The two volumes under review here provide useful treatments of two aspects of this collection of ideas. It is characteristic that although both books are in principle devoted to mechanics in general, they are in fact largely concerned with fluid mechanics. The essential nonlinearity of the governing equations has forced workers in fluid mechanics to pioneer in the exploitation of the ideas of similitude.

The first layer to be peeled from the onion of similitude is almost always that of dimensional analysis. To this 'art full of dangers' Saint-Guilhem attempts to bring in his little monograph a fresh and logical approach. After illustrating with several physical examples the paradoxes and errors that may arise from conventional schemes of dimensional analysis, he develops a mathematical theory of the invariance of elements in vector space under the group of affine transformations. This formalism is then applied to deduce a general theory of dimensional analysis, which has the virtue of providing the maximum amount of information that can be drawn from considerations of homogeneity. Various examples illustrate that this optimal goal is not invariably attained using the theorem of Vaschy (which, the author waspishly insists, has priority over the Buckingham pi theorem more familiar in English-speaking countries).

Birkhoff has emphasized the value of distinguishing between *dimensional* *analysis*, where one merely supposes that some relationship exists between a number of physical quantities, and *inspectional analysis*, where the actual equations governing the phenomenon are known. Saint-Guilhem, without recognizing this distinction, is primarily concerned with the latter situation, which often permits more specific conclusions. When applied to physical similitude, for example, his theory allows models that are distorted (not geometrically similar), and so verges on the subject of similarity rules. One might hope that if this concise and elegant discussion is not made widely available in translation, its ideas—together with those of Birkhoff—will at any rate be incorporated in future English and American treatments of dimensional analysis.

Sedov's monograph has been for twenty years the bible for a whole school of research workers in the Soviet Union. It begins with a treatment of dimensional analysis that is decidedly conventional in comparison with Saint-Guilhem's. The reader will (despite differences in notation and basic units) profit from a comparison of these two authors' treatment of the famous controversy between Rayleigh and Riabouchinsky regarding the dimensionless law for forced convection in an incompressible stream. A number of straightforward applications of dimensional reasoning are made to fluid-mechanical problems, including the author's own researches on water planing, impact, and surface waves, and viscous flow and turbulence.

The heart of Sedov's book is a long chapter—almost half the pages—concerned with one-dimensional unsteady motion of a gas. It gives a comprehensive treatment of problems that can be reduced to ordinary differential equations, the motion being *self-similar*. (This term is happily gaining general acceptance as the English equivalent of the Russian *avtomodel* and the German *homolog.*) The best known example is the blast wave or point explosion, which was integrated numerically by G. I. Taylor, and in closed form by Sedov, using the methods expounded in this book. However, the prototype of such solutions —Guderley's analysis of converging cylindrical and spherical shock waves (*Luftfahrtforsch.* **19**, 1942, 302)—is unaccountably not mentioned. The technique of solving these problems by topological analysis in the phase plane is clearly and amply explained. Any worker concerned with a problem that might possess self-similar solutions is well advised to turn to this chapter for enlightenment. The same theme is continued in the final chapter, with applications to astrophysical problems.

The translation is generally good. A number of typographical errors in the last Russian edition have been corrected; and the new ones are minor, except that on pages 170, 171, and 173 the translators have mistakenly replaced the expression $p/(\gamma-1)\rho$ for the internal energy of a perfect gas by the expression $\gamma p/(\gamma-1)\rho$ for its enthalpy.

M. VAN DYKE

Mechanics of Solids and Fluids. By Robert R. Long. Prentice-Hall, 1961. 156 pp. £2. 5*s*.

Fluid Dynamics. By G. H. A. Cole. Methuen, 1962. 238 pp. £1. 5*s*.

Text-books on fluid mechanics seem to be coming thick and fast at the present time. It is as well that they are, since developments in the understanding of old parts of the subject and the rapid growth of entirely new parts have left most university syllabuses looking rather dusty. What is needed is not simply a set of books on the advancing frontier of fluid dynamics, but a reconsideration of the older parts that have formed and will continue to form the foundation of a university course. Basic material selected and presented from a modern point of view is what every university teacher wants, and despite the supply of new text-books he has difficulty in finding it. This is a common lament, in schools of both engineering and applied mathematics.

Professor Long's little book is aimed at theoretically-minded engineers, and makes a useful contribution to student literature as a vehicle for instruction in the use of Cartesian tensors. His book has arisen out of a course of lectures on elementary mechanics of solids and fluids, given to third- and fourth-year students of engineering science, and it benefits in clarity and directness from so specific an origin. The primary purpose is to give what the preface calls a 'rigorous and plausible development of the equations of motion of elastic solids and Newtonian fluids'. I think he succeeds in revealing the common features of the analysis of these two kinds of media, and does so economically, so that the reader is left in a better position than if he had separate accounts of solids and fluids. However, the book is so short that it hardly does more than establish the equations of motion; and it is doubtful if a student is left with sufficient experience or understanding of what fluids and solids are likely to do in conformity with those equations. The author no doubt intends other books to be used as a sequel, but it may happen nevertheless that the rather arbitrary end-point of his book gives readers a false impression; there are already too many books, mostly written by and for physicists, which suggest that fluid mechanics has been covered once the equations of motion have been derived.

A subsidiary purpose of the book is to introduce Cartesian tensors as a standard working tool for engineering undergraduates. Here I think the author has succeeded admirably, and my only comment is that I think he might have gone further in this mathematical direction and described the vector differential operators (grad, div, curl) in general terms, without reference to particular co-ordinate systems, and the various integral transformation theorems, such as Stokes's theorem. Quite apart from the direct practical value of relations which can be adapted to any system of orthogonal curvilinear co-ordinates, there are real pedagogical advantages in the early introduction of the integral transformation theorems. For lack of them, the author is obliged to derive his mass and momentum equations by the unsatisfactory device of considering a parallelepiped of material. Matter does not come in units of this or any other shape; and vector methods do the job for an element of arbitrary shape both more neatly and (as in the proof of symmetry of the stress tensor) more rigorously.

The plan of the book gives an accurate impression of its purpose and character. The first chapter on vectors and tensors lays the mathematical foundations with a clear and concise account of the elementary properties of Cartesian tensors. Chapter 2 gives the analysis of stress in a continuous medium, and chapter 3 the analysis of strain and rate of strain near any point. The fourth chapter, regrettably short, is a very welcome description of the mechanical properties of materials, both solid and fluid; I read this with pleasure and hope that sometime the author will expand it up to a small book which can be put in the hands of students, especially applied mathematicians, who wish to know both the facts about materials in general terms and qualitative explanations of the relation between the macroscopic properties and the microscopic structure. Chapters 5 and 6 then turn to solids, and give the equations of motion and the solutions of some simple problems in elasticity. Chapters 7 and 8 do the same for fluids; here the author's touch is less sure, and one might quarrel with his choice of material to illustrate the use of the equations of motion. A rapid skip through hydrostatics, the equations for an inviscid fluid in Lagrangian form, Cauchy's vorticity equations, Bernoulli's equation, and the Blasius solution for the boundary layer on a flat plate, all in 18 pages, will give an undergraduate a strange impression of what fluids do, as well as indigestion, although no selection of material to suit this space can be wholly satisfactory.

The group of readers at whom Dr Cole's book is directed cannot be inferred so readily from the preface and contents list; but, whatever the target, it has not been hit. The book (one of the well-known series of Methuen Monographs on Physical Subjects) starts from first principles, and is laid out in the manner of a text-book on fluid dynamics for applied mathematicians. There are chapters on the equations of motion of an inviscid fluid, irrotational flow in various contexts such as surface waves and sound waves, the Navier–Stokes equation, and the laminar boundary layer; less conventionally, these are followed by chapters on thermal effects, dimensionless parameters, 'turbulent conditions', and hydromagnetics. The list of chapter headings thus has an up-to-date look, and covers most of the topics, except gas dynamics, which one might hope to be able to include in an advanced undergraduate course. So far so good; and that is about as far as it is good. For although Dr Cole has respect for modern developments, and a reputable publisher, he does not always know what he is talking about. Howlers, misunderstandings and careless statements are numerous, and destroy one's confidence in any part of the book. Many of these misunderstandings are at such a fundamental level as to appear to be incompatible with systematic teaching or research in fluid dynamics by the author, and I can suppose only that he has bravely but unwisely written a book before becoming properly acquainted with the subject. Here are a few examples which speak for themselves:

(1) In the Introduction, in the course of remarks about the properties of fluids, the author says 'The appearance of tangential stresses is associated

with the action of dissipative forces (such as viscosity) which oppose the motion to which they are due. If the fluid motion is very small the effect of the dissipative forces is small because the tangential stresses themselves are small'.

(2) On page 18, the equation for vorticity $\boldsymbol{\omega}$ in an inviscid incompressible fluid is found to be

$$\frac{D\boldsymbol{\omega}}{Dt} = \boldsymbol{\omega}.\nabla\mathbf{v}, \tag{1·45}$$

with conventional notation. The following paragraph reads:

'Any situation for which the right-hand side [of this equation] vanishes identically is of especial interest since then the total derivative of the vorticity is zero. This means that for such fluid flow any initial condition of vorticity is maintained throughout the motion. In this connexion an important result for incompressible fluid flow follows immediately from (1·45). If $\boldsymbol{\omega}$ is zero at any point on a streamline at any given time it is zero everywhere along the streamline, for from (1·45) it follows that if $\boldsymbol{\omega} = 0$ at any point on a streamline then $D\boldsymbol{\omega}/Dt = 0$ at all points along the streamline.'

(3) On page 23, after remarking that the circulation round a closed curve in the fluid is zero in irrotational flow [and without adding the qualification that the closed curve must be reducible], the author goes on: 'This means that $d\phi = \mathbf{v}.d\mathbf{l}$ is a perfect differential, and closed streamlines cannot occur in irrotational flow.' The next sentence is also quotable: 'A fluid in irrotational flow, such that the fluid velocity on each plane perpendicular to the direction of motion is constant, will often be referred to as being in plane flow.'

(4) Vorticity seems to be an especially weak spot in the author's education. On page 79, and immediately after obtaining the equation for $\boldsymbol{\omega}$ in a viscous fluid (viz. (1·45) above with the additional term $\nu\nabla^2\boldsymbol{\omega}$ on the right-hand side), this is his discussion of the important fact that viscous forces generate vorticity at boundaries: 'Thus even when $\boldsymbol{\omega} = 0$, $D\boldsymbol{\omega}/Dt \neq 0$; a viscous fluid may ultimately contain vortex flow (which moves in from the boundaries with a finite velocity) even though it was initially in irrotational flow. For incompressible flow variations of the velocity due to boundary changes are transmitted instantaneously through the fluid, as distinct from the vorticity which still has a finite velocity'.

(5) Page 120. 'G [the Grashof number] is a measure of the relative importance of the buoyancy and viscous forces acting on the fluid.'

(6) About two-dimensional flow past a blunt body with an axis of symmetry, we read, on page 157: 'There will be a stagnation point on the upstream side of this axis: for an ideal fluid this will be the point X on the surface but for a real fluid the stagnation point will be upstream of the surface to some extent, due to the presence of the boundary layer.'

(7) Page 183. 'Turbulent flow is rotational; because the fluid is always continuous the streamlines must take circular paths forming eddies and this is observed to be the case in practice.'

These are not isolated lapses, and one could go on having fun at the author's expense; for instance, there are errors, none of them trivial, in the streamline patterns in figures 2, 5, 9, 12, 13, 16 and 19. The book is simply too full of errors to be worth contemplation as a text for students. The mathematical details of standard solutions for particular flow fields are mostly given correctly, but the selection, ordering and interpretation of the material reveal the same lack of understanding as the above quotations and need not be commented on further. No; mention of turbulence, m.h.d. and thermal effects may stamp the author as being forward-looking, and in touch with current trends, but there is really no substitute for the old-fashioned virtues of understanding a subject and writing about it with care.

G. K. BATCHELOR

Journal of the Atmospheric Sciences. Bimonthly, published by the American Meteorological Society. \$18.00 (foreign subscribers, \$19.00) per volume of six parts.

Journal de Mécanique. Quarterly, published by Gauthier-Villars. 60 NF (foreign subscribers 70 NF, or \$14.00) per volume of four parts.

Fluid dynamicists will be interested to know of the existence of these two new journals.

The *Journal of the Atmospheric Sciences*, which is edited by R. Jastrow and N. A. Phillips, is a new expanded form of the former *Journal of Meteorology*, and is to provide a medium for publication of scientific papers concerned with the atmosphere of the earth and other planets. The declared editorial policy will be 'to encourage papers which emphasize the quantitative and deductive aspects of the subject', by which is meant, presumably, that the editors would like to receive papers of this kind. Major topics in the journal are expected to be: atmospheric structure and dynamics; radiation; cloud physics and chemistry; electrical properties; interaction with the oceans; extension to the interplanetary plasma; upper atmosphere phenomena produced by solar activities, including aurorae, airglow and ionospheric absorption; atmospheric effects of magnetically trapped particles; the origin and evolution of planetary atmospheres. Manuscripts for publication should be sent to the editors of the journal, at the American Meteorological Society, 45 Beacon Street, Boston 8, Massachusetts, and subscriptions should go to the same address.

The *Journal de Mécanique* seems likely to meet a long-felt need in French science for a publication containing papers of medium length. The editors are P. Germain, L. Malavard and R. Siestrunck, and there is a large international editorial committee of great distinction although, one suspects, with little real responsibility for the journal's future. The preliminary pages record that the journal was founded by Professor J. Pérès, who died in February 1962, just one month before publication of the first number. Judging by the contents of the first number, papers will range over the mechanics of fluids, of solids and of particles, and will be concerned with physical aspects as well as with the traditional 'rational mechanics'. Papers for publication should be sent to the editors, at Institut Henri Poincaré, 11 rue Pierre Curie, Paris 5e, and subscriptions should be sent to the publishers at 55 Quai des Grands-Augustins, Paris 6e.